The role of Deductive Reasoning on the (Theoretical) Computer Science research

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Abstract. The *methodology of scientific research* with special attention to the specificity of the contribute of Logic and Mathematics in the *Computer Science* is approached. The localization of the Theoretical Computer Science on the field of science disciplines is discussed and the *deduction* is identified as dominating research method on this field. In the second part of the work, it is suggested the use of syntactical proofs performed in abstract deduction systems to assist the verification of proofs on Theoretical Computer Science. This procedure is useful to prevent the traditional errors resultant of the weak rigour used on the structuration of the mathematical proofs.

1 Motivations

With this work we intend to reflect on some aspects that identify and distinguish the research in Logic and Mathematics of the research done in other scientific areas, particularly regarding the procedures and tools used. This interest has appeared during the choice of the theme from the topic list of the call for paper of the MAP-i advanced seminar.

In this context, we felt some difficulties in choosing it, because in order to do it we should clarify some ideas, such as, how to differentiate method and methodology in the field of scientific disciplines. For obvious reasons, we chose the topic *theoretical research*: firstly, our scientific background is in Mathematics where the objects are of theoretical nature and dominantly abstracts. Secondly, our doctoral project is in the area of Theoretical Computer Science, being hence, a mathematical project. Therefore, we wanted to deal with our unknowledgement and consequent discomfort to study these ampler discussions of the modern science, which place the Logic and Mathematics in a very particular position of the Science field. In this context, we looked for answers to questions like "What is the Science?", "What does differentiate methodology of the method on scientific investigation?", "Is there a methodology that can assure the rigour and the scientific statute of all the knowledge areas?" and "In what sense can we consider the deductive method also as a scientific methodology?".

1.1 Method and Methodology: logic and procedure

In order to understand the questions "What is a methodology?" and "What is the Scientific Method?", it is necessary to answer to another deeper question studied by the Philosophy of Science : "What is the Science ?".

Typically, *Science* is defined as:

> (...) (knowledge obtained from) **the systematic study** of the structure and behaviour of the physical world, especially **by observing, measuring and experimenting, and the development of theories** to describe the results of these activities: pure/applied science, recent developments in science and technology, Space travel is one of the marvels wonders of modern science.(...)
>
> from *Cambridge Advanced Learner's Dictionary*

These practices of observation, measurement and experimentation are strictly related to the concept of *scientific method*, expressed in the following: [1]

1. Observation and description of a phenomenon or group of phenomena.
2. Formulation of a hypothesis to explain the phenomena. In physics, the hypothesis often takes the form of a causal mechanism or a mathematical relation.
3. Use of the hypothesis to predict the existence of other phenomena, or to predict quantitatively the results of new observations.
4. Performance of experimental tests of the predictions by several independent experimenters and properly performed experiments.

This is the hypothetic-deductive method that is very useful to build the knowledge in fields of experimental nature as the natural sciences (eg. physic and chemistry). In the past, the use of this method was seen as a criterion to decide which knowledge fields are (or not) a science (cf. [1]). In fact, this method was essential to the affirmation of some new sciences as sciences (eg. Psychology). However, this criterion shows to be excessively strong to characterize what is or not science. For instance, it is impossible to repeat some social phenomena to build or validate a theory. Moreover, as we will see, this method does not characterize the reasoning in some theoretical research investigations. To understand the scientific knowledgement as the result of a practice based on a specific and rigorous method, allow us to assume the scientific production, just as the application of a concrete set of rules defining how to proceed in posing new relevant questions and formulating successful hypotheses. It is not exclusive of the hypothetic-deductive method. To define *Science* strictly by its logic of procedures can make the Mathematics and Logics invisible as a science, or reduce it just to its instrumental role in the validation of empirical proofs of another knowledge areas, in which, its science statute, was achieved by the use of the hypothetic-deductive method. As we see in Figure 1, the nature of objects of

[1] Actually, there are in the literature several variations of this concept. This one is defined in [2] and, for another characterisation, we suggest eg. [1];

Mathematics and Logic is abstract (propositions, relations, etc.) and therefore it can not be observed, measured or experimented, as basis of development and validation of theories.

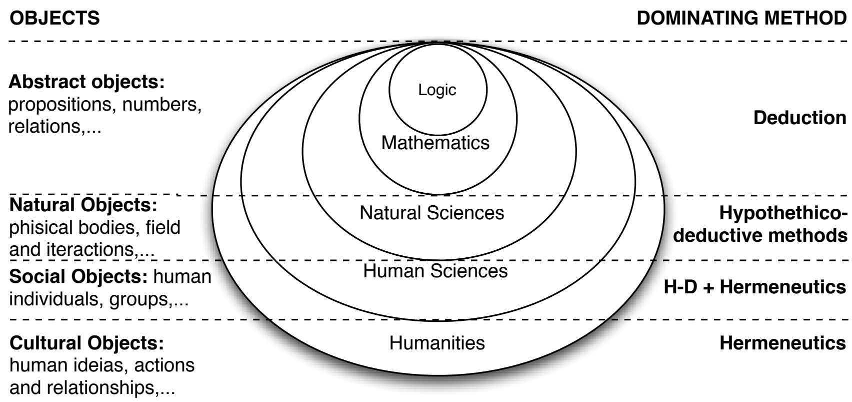

Fig. 1. Sciences, objects and methods (adapted from [1])

Actually, according to *Sandra Harding*:

> "a research *method* is a technique for (or way of proceeding in) gathering evidence"

In this sense, the validation of scientific knowledge has to be done in a more ample perpective, considering the theory, the way by which the investigation is done and the nature of the produced knowledge, i.e., has to follow a methodology:

> "*methodology* is a theory and analysis of how research does or should proceed" *Sandra Harding*

In order to localize Mathematics in this discussion, we cite the famous mathematician *Jean Dieudonne* that argues:

> Logical Deduction ... is the one and only true powerhouse of mathematical thinking.

Bearing in mind these considerations it is suggested in [2] that Science is:

> (...) the field of study which tries to describe and understand the nature of the universe in whole or part. The field of study or discipline that we call Science is spelled with a capital "S" as it is a proper noun in this use while science with a small 's" is the application of this discipline.(...)

1.2 What is *Computer Science*?

Returning to the same dictionary, it is stated that *Computer Science* consists in:

> the study of computers and how they can be used.

However, despite this simplicity, the definition of Computer Science is by itself, a non consensual subject within Computer Science community. This difficult results of its huge field range, being by this reason considered by some authors as a set of sciences. Actually, following [5], the discipline of Computer Science is split in a set of 14 sub-areas. The research of these sub-areas is also covered by a considerable range of scientific fields, such as, mathematics (eg. *Discrete Structures*, *Programming Fundamentals* and *Algorithms and Complexity*), classical engineering (eg. Software Engineering), human sciences (Artificial Intelligence), etc. Hence, it is natural the necessity of the use of different methods and methodologies to research in Computer Science, in function of the topics and sub-areas in study. For instance, in order to investigate properties of the behaviour of a particular software systems, it can be adequate the adoption of the hypothetical deductive method, in order design a large software system, it can be adequate the engineering method, etc..

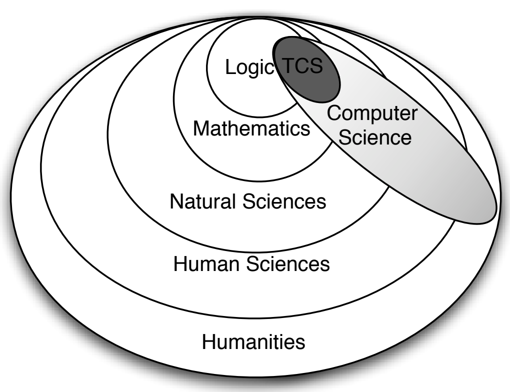

Fig. 2. Position of Computer Science in the Science

1.3 How can we research on Theoretical Computer Science?

In order to answer the stated question, it is necessary to circumscribe the area of Theoretical Computer Science in Computer Science once, such as in the case of

Computer Science, this definition is not consensual and their bonds are not clear. Despite this discussion, we consider as Theoretical Computer Science field, the collection of topics of Computer Science related with the theory of computability, analysis of algorithms and semantics of programming languages. As far as the research methods are concerned, it is used the classical method of doing mathematics: conjecturing and demonstrating, i.e., the *deductive method*. According to [1]:

> (...) Concerning Theoretical Computer Science, which adheres to the traditions of logic and mathematics, we can conclude that it follows the very classical methodology of building theories as logical systems with stringent definitions of objects (axioms) and operations (rules) for deriving/proving theorems. (...)

In the context of the Science, the topics of Theoretical Computer Science fit in the Logical and Mathematics fields (cf. Figure 2) where, actually, the dominating research method is the *deductive method* (cf. Figure 1).

2 Assisting deductive proofs with abstract deductive systems

The discipline of Mathematics plays a central role in the Science, once it is used as supporting language of the scientific knowledge representation. This representation explores the descriptive role of this science (eg. by statistical analysis of data). However, Mathematics is in its essence a purely deductive science:

> (...)Mathematics is understood only in its descriptive role in providing a language for scientific, technical, and business areas. **Mathematics, however, is really a deductive science**. Mathematical knowledge comes from people looking at examples, and getting an idea of what may be true in general. Their idea is put down formally as a statementa conjecture. The statement is then shown to be a logical consequence of what we already know. The way this is done is by logical deduction (...)
>
> In[7]

We approach in this section the logico-mathematical reasoning in its deductive perpective. As observed in the last section, the Theoretical Computer Science follows, like in other theoretical research fields, the deduction as a dominating research method. Being true that this method, by itself, assures the validation of the deduced results, it is usual to find proofs with some errors resultant of the violation of the method. These violations are caused by the typical human errors like forgetfulness and distractions. To avoid these problems is essential to follow a rigorous structuration of proofs and the *abstract deductive systems* studied on mathematical logic offer a good tool to assure it. An *abstract deductive system* is a formal system used to reasoning in a rigorous way deriving an expression from one or more other expressions antecedently expressed in the language of that system. These expressions may represent a description of modeled phenomena (i.e. *semantics*) but, their derivations should be done following exclusively the structure of inference of the system (i.e. in a pure *syntactic* way). By this reason and in order to distinguis these proofs from the "general" mathematical proofs (developed at the semantic level), we call them by *syntactical proofs*. Now, the concepts of syntactical proof and deductive system are formalised: the more traditional way to define an *abstract deductive system* is considering a pair $\langle \Phi, \mathrm{IR} \rangle$ where Φ is a set of axioms and IR a set of inference rules. In this context, given a system $\mathcal{H} = \langle \Phi, \mathrm{IR} \rangle$ and a set of formulas Γ we say that ϕ has a *syntactical proof* from Γ in $\mathcal{H}$, in symbols $\Gamma \vdash_{\mathcal{H}} \phi$, if there is a finite sequence of formulas $\phi_1, \ldots, \phi_n$ such that ϕ_n is ϕ and for every $i = 1, \ldots, n$ one of the following conditions holds:

- $\phi \in \Phi$;
- $\phi \in \Gamma$;
- there is an inference rule $\frac{\psi_1 \cdots \psi_k}{\phi_i} \in \mathrm{IR}$ such that, for any $r \in \{1, \ldots, n\}$ $\psi_r = \phi_l$ for some $r \in \{1, \ldots, k\}$.

As stated above, the intrinsic structuration of syntactical proofs can be used as a tool to assert the correction of deductive proofs, avoiding errors of reasoning. In this context, we can define a procedure to confront in a systematic way, a "proof candidate" of a given conjecture with its "respective syntactical proof", i.e., to confront the deduction to verify with a syntactical proof obtained by the deduction of the characterization of this conjecture in an abstract deductive system. This procedure is illustrated in the Figure 3 and is summarized in the following steps:

1. choice an adequate abstract deductive system;
2. characterize the conjecture in this system;
3. perform the syntactical proof;
4. make the semantical interpretation of the resulting proof;
5. confront the result with the "candidate proof";
6. validate or not the deduction.

Note that the adequate choice of the deductive system is essencial to allow these verifications. This choice may be done in function of the nature of the "candidate proofs" and of the experience and expertise of the researcher. As illustrative example, we perform a verification of a trivial deduction, using the (propositional version of the) *natural deduction system*. Natural deductive systems are characterized by the absence of axioms. Its set of inference rules is split in two kind: rules of introduction and rules of elimination. As examples of introduction rules we have

$$\frac{\varphi \qquad \psi}{\varphi \wedge \psi} \wedge \mathrm{I}$$

and of elimination,

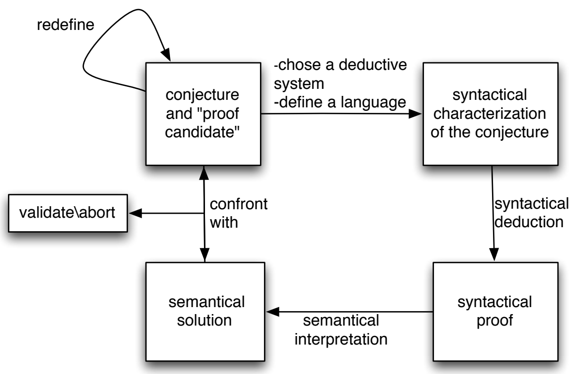

Fig. 3. Scheme of the validation of proofs

$$\frac{\begin{array}{cc}\vdots & \vdots\\ \phi & \phi \rightarrow \psi\end{array}}{\psi} \rightarrow \text{E.}$$

The complete presentation of this deductive system can be foud, for example, in [6]. Following the previous procedure, we should identify the atomic parts of the reasoning as propositional variables and interpret the logical connectives $\{\neg, \rightarrow, \wedge, \vee, \perp, \top\}$ as our logical negation and implication, our "and" and "or" and, our "false" and "true" respectively (cf. [4]). For instance, let's consider the simple deduction: *"Supposing that I am martian and that the martians live in Mars, we conclude that I am martian and that I live in Mars"*. Firstly it is necessary to identify its atomic propositions:

- p - *"I am martian"*;
- q - *"I live in Mars"*.

Looking to these atomic propositions we can express the result of that deduction as $\{p, p \rightarrow q\} \vdash p \wedge q$. Now we use the *natural deduction* system to verify its correction:

$$\frac{\dfrac{p \rightarrow q \qquad p}{q} \rightarrow E \qquad p}{p \wedge q} \wedge \text{I}$$

It is important to note that these syntactical proofs are really intuitive. For example, if we replace in the previous proof the propositional variables for its atomic propositions (represented in red), we obtain the easily readable expression:

Suppose that if I'm martian then I live in mars. Suppose also that I'm martian
Then we conclude that I live in Mars — *supposing again that I'm martian*
we conclude that I'm martian and that I live in Mars

Actually, the motivation of the definition of these systems was its proximity with the human reasoning. This aspect is a characteristic of the natural deductive systems class, i.e., of the class of systems with this definition "style" (concerning their inference structure) but possibly defined with other languages, (eg. first order, higher order etc) and for other reasoning paradigmes (classical reasoning, intuitionistic reasoning, paraconsistent reasoning, etc.). Regarding to the presented example, the abstract deductive system is manifestly poor and shows with insufficient expressivity to deal with real (or at least interesting) proofs. In fact, it just uses a propositional language which is one of the most simple languages used in mathematical logic. Fortunately, there are another versions of this deductive system that use more expressive logics (eg. *Natural Deduction in First Order Logic* cf. [6] or [3]). Moreover, there are also the Hilbert-Style deductive systems[2] and versions of the presented system which performs classical reasoning. Syntactical proofs performed in these systems are few natural when compared with the human reasoning, and deals with complex expressions resulting of instantiations of long axioms. For instance, the proof of the simple proposition $\varphi \to \varphi$, in natural style result of one deduction step and, in the Hilbert style, result of a deduction with five steps including expressions like $(\varphi \to (\phi \to \psi)) \to ((\varphi \to \phi) \to (\varphi \to \psi))$ (cf. [3]). Hence, this style of systems are not a good choice to the intended goal. However, they can be useful if we just want to verify the validity of conjectures (instead of verifying the structural correction of the developed "candidate" of proof). Returning to the language choice, being true that higher level languages offer a more expressive power to deal with real deductions, the complexity of performed syntactical proofs in these systems is highly increased, what makes this method more hard and, sometimes, impracticable. In order to minimize this problem, we suggest the interest of explore the use of *interactive theorem provers* and *automatic proof assistant* tools like `coq` to assist this work. Traditionally, this automatic assistants are based on *Curry-Howard isomorphisms* using the intuitionistic paradigme (cf. [3] or [6]). Commonly, mathematical reasoning makes use of the principle of the *excluded middle*[3] which is not valid in these systems. This principle allows the traditional proofs by *reduction ad absurdum* and distinguishes the *classical* of the *intuitionistic reasoning.* Fortunately, by introducing some inference rules, we can adapt these systems to produce classical reasoning, which enable the use of these automatic tools to the intended propose.

3 Conclusions

The present work is an essay, in which it is discussed a personal perspective about the use of deductive systems as a tool to assist the verification of mathematical proofs in development of Theoretical Computer Science research. In the first part

[2] Usually, logical systems that have few inference rules based on an extensive list of axioms are known by *Hilbert-style proof systems.* They are the most traditional presentation of deductive systems;

[3] Excluded Middle: *"Every proposition is either true or false."*

it was discussed the *methodology of scientific research* with special attention to the specificity of the contribute of Logic and Mathematics to the *Computer Science*. In this view, the position of the Theoretical Computer Science in the Science field is stated and the deductive method is identified as the dominating method to the research in this field. In this context there were answered questions like "What is the Science?", "What does differentiate the methodology of the method on scientific investigation?", "Is there a methodology that can assure the rigour and the scientific statute of all the knowledge areas?" and "In what sense can we designate the deductive method also as a scientific methodology?".

In a second moment of the paper, the concepts of syntactical proof and formal deductive systems were formalized. In this context it was suggested the use of syntactical proofs to verify the correction of deductions made *à priori*. It was also made a brief discussion about some particular choices of deductive systems, suggesting as a good choice, their *natural style* versions.

The text was developed to general readers not necessarily familiarized with *formal logic* and, in this context, we tried to write a text understandable to a more general public leaving the more technical discussions outside of the work scope. Hence, as further work it would be interesting to produce a deeper discussion about the choice of the deductive systems covering aspects as the decidability, illustrating the procedure with a more realistic examples.

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